MULTIMEDIA UNIVERSITY

STUDENT ID NO

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2019/2020**

## TMA1301 – COMPUTATIONAL METHODS

(All sections / Groups)

4 MARCH 2020
9.00am – 11.00am
(2 Hours)

---

**INSTRUCTIONS TO STUDENT**

1. This question paper consists of **4 pages** with **3 questions** only.
2. Attempt **ALL THREE** questions. The distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided and start each solution of a question on a **new page**.
4. Show all steps.

## QUESTION 1 ( 5 MARKS)

Given the function $f(x) = \dfrac{\sin x}{x} - \cos x$.

(a) Rewrite the given $f(x)$ to avoid loss of significance by using the first two nonzero terms in Taylor series expansion. [2.5 marks]

[HINT: $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots$; $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \ldots$; $\tan x = x + \dfrac{x^3}{3} + \dfrac{2x^5}{15} + \ldots$]

(b) Calculate the approximated value of $f(0.25)$ by using **SIX DIGIT arithmetic** with rounding. [1 mark]

(c) The actual value of $f(0.25)$ is 0.02070. Find the relative error from the result obtained in (b). Correct your answer into **FIVE decimal places.** [1.5 mark]

**Continued...**

## QUESTION 2 ( 15 MARKS)

(a) Consider the function $f(x) = x^3 - 5x^2 + 7x - 3$.

(i) Find $f'(x)$ and $f''(x)$. [1 mark]

(ii) Using Newton's method, find a root of $f(x)$ starting with $p_0 = 0.9$ by completing the following table. Use **FOUR decimal places** for all workings.

[3 marks]

| Iteration, $n$ | $p_n$ | $f(p_n)$ | $f'(p_n)$ |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |

(iii) Given the root of $f(x)$ is $r = 1$, is the convergence linear or quadratic as Newton's method is used to find the root? [1.5 marks]

(iv) Based on the answer obtained in (iii), find the convergence rate. Hence express the error $e_{n+1}$ in terms of the previous error $e_n$.

[1.5 marks]

(b) Consider the definite integral $\int_2^3 \frac{1}{x} dx$. Use **FOUR decimal places** for all workings.

(i) Find the actual value for the definite integral. [1.5 marks]

(ii) Find the number of subintervals if Composite Trapezoidal Rule is used to approximate the definite integral with an error of at most $2 \times 10^{-3}$.

[Hint: error formula : $-\frac{b-a}{12} h^2 f''(x)$] [4.5 marks]

(iii) With the number of subintervals obtained in (ii), find the approximated value for the definite integral using the Composite Trapezoidal Rule.

[2 marks]

**Continued...**

## QUESTION 3 (20 MARKS)

(a) Consider the following linear system:

$$\begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

(i) Use row reduction technique to find an upper triangular **U** and lower triangular **L** in the **LU factorization** of the given linear system. [2.5 marks]

(ii) Then find the values of $x_1$, $x_2$ and $x_3$ by using the **L** and **U** obtained from (i). [3 marks]

(b) Given the following system of equations:

$$7w - 2x + y + 2z = 3$$
$$2w + 8x + 3y + z = -2$$
$$-w + 5y + 2z = 5$$
$$2x - y + 4z = 4$$

(i) Construct the equations for $w$, $x$, $y$ and $z$ of the above linear system using Jacobi Method. [2 marks]

(ii) Copy the following table into your Answer Booklet. Complete it by computing one iteration of the Jacobi Method for the constructed equations in (i) starting with $[w, x, y, z] = [0, -1, 1, 1]$. Write your answers correct to **FOUR decimal places.** [2 marks]

| $n$ | $w$ | $x$ | $y$ | $z$ |
|---|---|---|---|---|
| 0 | 0 | -1 | 1 | 1 |
| 1 | | | | |

(c) Determine whether $\lambda = -7$ is the eigenvalue of matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. [1.5 marks]

(d) Consider the following data:

| $x$ | $x_0 = 1$ | $x_1 = 2$ | $x_2 = 3$ |
|---|---|---|---|
| $f(x)$ | 3 | 5 | 4 |

(i) Construct the Lagrange Coefficients $L_0(x)$, $L_1(x)$ and $L_2(x)$. [1.5 marks]

(ii) Find the second interpolating polynomial $P_2(x)$ using the results from (i). [1 mark]

(iii) Approximate the value of $f(1.5)$ from the $P_2(x)$ obtained in (ii). [0.5 mark]

**Continued...**

(e) The owner of a motor workshop keeps track on number of cars repaired and the revenue obtained. Based on a sample data, he has the following information:

| Number of cars repaired | 25 | 10 | 15 | 12 | 16 | 24 | 20 | 18 | 10 | 20 | 20 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Revenue(RM'000) | 8 | 3.5 | 6.5 | 6 | 7.5 | 9 | 7.8 | 8 | 6.8 | 7 | 6.5 | 7 |

(i) Copy the following table into your Answer Booklet and complete it.

| $x$ | $y$ | $x^2$ | $xy$ |
|---|---|---|---|
| 25 | 8 | | |
| 10 | 3.5 | | |
| 15 | 6.5 | | |
| 12 | 6 | | |
| 16 | 7.5 | | |
| 24 | 9 | | |
| 20 | 7.8 | | |
| 18 | 8 | | |
| 10 | 6.8 | | |
| 20 | 7 | | |
| 20 | 6.5 | | |
| 18 | 7 | | |
| $\sum x =$ | $\sum y =$ | $\sum x^2 =$ | $\sum xy =$ |

[2 marks]

(ii) From (i), find the equation of the best fit linear line $y = a + bx$ that models the data by using the least squares method. Round your answers to **FOUR decimal places.**

[Hint: $a = \dfrac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$, $b = \dfrac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$]

[3 marks]

(iii) From (ii), estimate the revenue (in RM) when 10 cars are repaired.

[1 mark]

**End of Page**